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## Constructing the Moduli Space of Elliptic Curves Over a Given Field $K$

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Studying a single instantiation of an object can be insightful, but it is often desirable to study families of the object at once. Choosing how one will identify "equivalent" objects and then choosing a suitable parameter by which to allow objects in the family to vary is what a classical moduli problem consists of. Elliptic curves have long been studied in geometry, but they received an increase in attention after connections with number theory were exploited due to their nature as abelian varieties. Typically, one wants what is known as a fine moduli space which solves the moduli problem and also carries a universal family that satisfies certain properties, but this is often not possible.

In this survey, I explain why this is not always possible, and offer the two common ways around this problem. One is to settle for what is called a coarse moduli space, which in the case of elliptic curves, leads to the notion of the modular curves $Y(\Gamma)$ and $X(\Gamma)$. The other option is to enlarge the category of schemes as one does with the enlargement of the category varieties to schemes. This leads to the notion of stacks like $\mathcal{M}_{1,1}$. The Survey concludes with new problems that arise in classifying elliptic curves up to isogeny rather than isomorphism, and fundamental questions about the geometry of these moduli spaces.