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Uniform Symbolic Topologies in Conic Monomial Rings

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Algebraic geometry (AG) and commutative algebra (CA) are major generalizations of linear algebra, fairly influential in mathematics. Since the 1980's with the development of computer algebra systems like Mathematica, AG and CA have been leveraged in areas of STEM as diverse as statistics, robotic kinematics, computer science/geometric modeling, and mirror symmetry. This talk will motivate the *ideal containment problem* in Noetherian rings that my dissertation attacks. A commutative ring R is Noetherian if every ideal I is finitely generated. Given a prime ideal P in a Noetherian commutative ring R, there are two ways to define the powers of P: regular vs. symbolic. If P is generated by n elements, the E^{th} regular power P^E of P is generated by up to nE elements in a manner similar to F.O.I.L. in K-12; thus algebraists find regular powers intuitive. Meanwhile, the symbolic powers $P^{(E)}$ of P are a collection of ideals--more technical to define, less intuitive--such that P^E is contained in $P^{(E)}$ for all integers $E>0$. When is it the case that for all positive integers $N>0$, there exists a positive integer $E>0$ such that $P^{(E)}$ is contained in P^N?

This ideal containment problem is quite hard and I attack it for conic monomial rings. Given any field K, a monomial ring over K is a K-vector space with ring structure, where the K-basis consists of Laurent monomials; a polynomial ring in finitely many variables over K is the simplest example. A monomial ring is *conic* if it can be constructed from a certain pairing of convex polyhedral cones in a Euclidean space--polynomial rings are conic. We show how to leverage basic data about these cones to solve a version of the containment problem: our solution is amenable to computer matrix algebra methods.